A NEW APPROACH TO IMPROVE CRITICAL AND CREATIVE THINKING SKILLS IN MATHEMATICS

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CRITICAL THINKING AND IMAGINATION

- Developing and improving students' critical thinking skills is a major goal of higher education.

- As educators, we should try to find various ways of teaching those skills in interactive, content-based and engaging way.

- This study introduces a new type of individual and team-based activities and games with the objective of improving students' critical thinking skills, imagination and creativity.

- As a byproduct, those activities will also strengthen students' knowledge in the respective mathematical topics.

"IN THE BEGINNING WAS THE WORD" (JOHN 1:1)

- It all started 6 years ago, when I was preparing for my next class which was Business Calculus

- It was not an exciting topic, so I started thinking about some type of exercises, which will be interesting and fun (why not?)

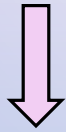- So, I came up with the following exercise:

SIX YEARS AGO …

- EVALUATE THIS EXPRESSION AND YOU WILL GET A THANK YOU IN A FOREIGN LANGUAGE.

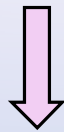$$\int dm \quad (ex)' \quad \left(\frac{r^2}{2}\right)' \quad \frac{c^{-3}}{c^{-4}} \quad \sqrt{-1}$$

⬇   ⬇   ⬇   ⬇   ⬇
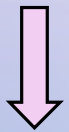
$$m \qquad e \qquad r \qquad c \qquad i$$

JUST EVALUATION

- It was an instant success!

- I also made an interesting observation: the student, who was the first to came up with the correct answer, was not one of my typical "A" students. (Later I noticed in several occasions, that the student has a sharp and creative mind.)

- Students liked the activity very much, it was engaging, and we were using this type of exercises frequently in our classes.

- Then we had a eureka moment:

   what if we ask students to solve the **reverse problem**, meaning, instead of giving them an expression and asking them to find the word, give them a word, give them specific *templates* for each letter, and ask them to create an expression using those templates which will generate the given word?

- Those templates with placeholders were later called "*mathlets*" which stands for "*mathematical letters*".

WHAT IS A MATHLET?

A **mathlet** is a new type of conceptual object.

It is a mathematical expression with placeholders

It generates **a single letter**.

The placeholders will be represented with the

symbol Δ and can be independently replaced wit

any **letter** or any **integer number**.

DIFFERENCE BETWEEN THE ORIGINAL AND REVERSE PROBLEMS

ORIGINAL (CALCULATION) - CALCULATE THE EXPRESSION

$$\int dz$$

Calculates the letter ⟶ z

Knowledge

EVERSE (CREATION)- REPLACE PLACEHOLDERS WITH LETTERS AND INTEGER NUMBERS IN SUCH A WAY THAT IT WILL GENERATE THE LETT

$$\int \Delta d\Delta$$

Replace the first placeholder with 1 and the second placeholder with z ⟶ $\int 1dz = z$

Knowledge & Critical Thinking

EXAMPLES OF MATHLETS FROM VARIOUS TOPICS

$$\sqrt[\Delta]{\Delta^\Delta}$$

$$\begin{vmatrix} \Delta & -4 \\ \Delta & \Delta \end{vmatrix}$$

$$\log_\Delta \Delta^\Delta$$

$$\Delta^{\log_\Delta \Delta}$$

$$\Delta\Delta \sin \Delta^0$$

$$\Delta\Delta \sin \Delta^0 \cos \Delta^0$$

$$\left(\frac{\Delta^\Delta}{\Delta} \right)'$$

$$\left(\frac{\Delta^\Delta}{\Delta} \right)''$$

$$\int \Delta d\Delta$$

$$\int_\Delta^{2\Delta} \Delta dx$$

$$\Delta^{-4}\Delta^\Delta$$

$$\frac{\Delta\Delta}{\sqrt{\Delta}}$$

WORKING WITH SINGLE MATHLETS

- We believe that even an exercise with a single mathlet will foster a critical thinking skills.
- **For example,** if we ask a student to evaluate the following expression: $\lg 10^a$, the student will use properties of logarithms and quickly come up with the solution, which is the letter "a". To do this, the student should have relevant knowledge, but none (or very little) critical thinking is involved.
- On the other hand, if we ask the student to generate the letter "a" using the mathlet $\lg \Delta^\Delta$ he/she should do some critical thinking, such as: "Well, in order to get an "a", one of those placeholders should be an "a". It cannot be the base since we need to get rid of the "lg", which is the common logarithm with the base "10". This implies that the placeholder in the base of should be "10" — the same as the base of the logarithm - we came up with $\lg 10^\Delta$

  The rest is straightforward and the answer is an "a"

- Example. Generate the letter "w" using the following mathlet: $e^{\int \Delta^{\Delta} d\Delta}$.

- Solution. In this mathlet, e is raised to an exponent which is an integral. Since the letter "w" should be generated, we have to get rid of "e". There are two ways of getting rid of "e".

  a. Making the exponent equals to zero. This will not work because we will end up with 1 instead of "w".

  b. Making the exponent equals to a logarithm with natural base (i.e., with base e). On the other hand, in order to get "w", the exponent, i.e., $\int \Delta^{\Delta} d\Delta$, could be not just any natural logarithm but lnw. And there is only one way to do it: $\oint w^{-1} dw = \ln w$. Thus, the answer is: $e^{\int w^{-1} dw} = w$

- As one can see, this exercise with a single mathlet required much higher level of critical thinking than the previous example.

USING MATHLETS – SOME EXAMPLES

Mathlet

$$\sqrt[\Delta]{\Delta^\Delta}$$

> Generate the Letter "g"

$$\sqrt[5]{g^5} = g$$

$$\log_\Delta \Delta^\Delta$$

> Generate the Letter "D"

$$\log_2 2^D = D$$

This process obviously improves critical thinking skills and imagination

$$\begin{vmatrix} \Delta & -4 \\ \Delta & \Delta \end{vmatrix}$$

> Generate the Letter "h"

$$\begin{vmatrix} h & -4 \\ h & -3 \end{vmatrix} = h \quad OR? \quad \begin{vmatrix} h & -4 \\ 0 & 1 \end{vmatrix} = h$$

$$\left(\frac{\Delta^\Delta}{\Delta} \right)'$$

> Generate the Letter "R"

$$\left(\frac{R^2}{2} \right)' = R$$

WORDS WITH MATHLETS

- Mathlets can be used in more advanced ways.

- For example, we can ask students to use mathlets to generate specific words or even short sentences. There are many variations of this type of exercises. For example, we may ask to create a specific word by
  - Using any (different from each other) mathlets from the provided list;
  - Using randomly selected mathlets;
  - Using specific mathlets, etc.

IT's YOUR TURN!

- EACH OF YOU RECEIVED AN ENVELOPE

- THERE ARE SEVERAL MATHLETS IN THE ENVELOPE

- TAKE OUT ALL THOSE MATHLETS AND USE THEM TO WRITE YOUR NAME☺

  - ATTENTION: ALL LETTERS IN YOUR NAME SHOULD BE GENERATED WITH DIFFERENT MATHLETS (EVEN IF YOU HAVE REPEATING LETTER IN YOUR NAME)

GENERATE THE WORD "Happy" WITH THE FOLLOWING MATHLETS

$$\log_\Delta \Delta^\Delta \qquad \Delta^{\log_\Delta \Delta} \qquad \frac{\ln \Delta^\Delta}{\Delta \ln \dfrac{1}{3}} \qquad \sqrt[\Delta]{\Delta^\Delta} \qquad \frac{\Delta^{-3}}{\Delta^\Delta}$$

$$\log_7 7^H \qquad a^{\log_3 3} \qquad \frac{\ln 3^p}{-1\ln \dfrac{1}{3}} \qquad \sqrt[3]{p^3} \qquad \frac{y^{-3}}{y^{-4}}$$

$$H \qquad a \qquad p \qquad p \qquad y$$

GENERATE THE WORD "Happy" – ANOTHER SOLUTION

$$\left(\frac{\Delta^\Delta}{\Delta}\right)'' \qquad \int_\Delta^{2\Delta} \Delta d\Delta \qquad \Delta\Delta\sin\Delta^0 \qquad \Delta^{-4}\Delta^\Delta \qquad \begin{vmatrix} \Delta & 2019 \\ \Delta & \Delta \end{vmatrix}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\left(\frac{H^3}{6}\right)'' \qquad \int_a^{2a} 1dx \qquad 2p\sin 30^0 \qquad p^{-4}p^5 \qquad \begin{vmatrix} 1 & 2019 \\ 0 & y \end{vmatrix}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$H \qquad\qquad a \qquad\qquad p \qquad\qquad p \qquad\qquad y$$

TEAM WORK AND COMPETITION WITH MATHLETS

- Mathlets can be successfully used in team works as well.
  - These types of activities enhance not only critical thinking but also team work skills.

- **<u>An Example</u>**. The instructor prepares a list of mathlets and a deck of words with the same length (e.g., each word consists of 4 letters).

- The class is divided into teams of 3 or 4 students.

- Each student gets a list of mathlets from the instructor and each team randomly draws a word from the deck of words.

TEAM WORK AND COMPETITION WITH MATHLETS (Cont.)

- The instructor specifies which mathlets can be used during the exercise.
  - For example, he/she can allow to use any mathlets from the list, or
  - use a specific subset from the list, or
  - Randomly select mathlets from the deck of mathlets
- The task of each team is to generate the word they have drawn with the condition that letters of the word should be generated with different mathlets.
- The team which completes the task first, wins

Thank You!

$$e^{\int \Delta^{\Delta} d\Delta} = e^{\int w^{-1} dw} = w$$

Thank You!